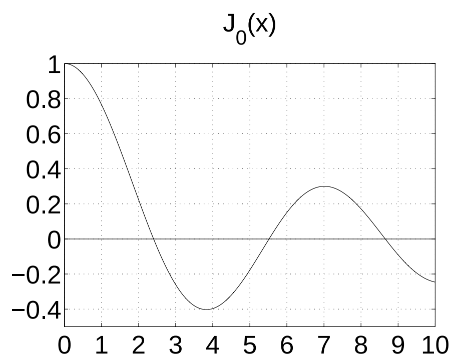
Project 1
Due on Fri, Nov 1

Use Matlab to solve problems below. You can use any numerical methods but you have to write your code from scratch. Since the main goal of this course is to learn algorithms, *you cannot use any black box*. That is, the Matlab functions such as `diff`, `fzero`, and `lu` cannot be used. Submit the results together with your Matlab codes by email. In the email, explain clearly what numerical methods you used for each problem.

1) The Bessel function $J_0(x)$ is a solution to $y'' + \frac{1}{x}y' + y = 0$ and behaves as shown in the figure. Find the first three zeros with at least five correct digits. For this problem, you can use the Matlab function `besselj(n,x)` to compute $J_n(x)$ and also use the property $J_0'(x) = -J_1(x)$ if necessary.



2) Consider the two-point boundary value problem,

$$-\varepsilon y'' + y = 2x + 1, \quad 0 \le x \le 1, \quad y(0) = 0, \quad y(1) = 0.$$

   (a) Show that the exact solution is $y(x) = 2x+1-\left(\sinh\dfrac{1-x}{\sqrt{\varepsilon}} + 3\sinh\dfrac{x}{\sqrt{\varepsilon}}\right)/\sinh\dfrac{1}{\sqrt{\varepsilon}}$.

   (b) Set $\varepsilon = 10^{-3}$. Make the mesh as $x_i = ih$, where $h = 1/(n+1)$. For $h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$, plot the numerical solution $w(x)$ together with the exact solution $y(x)$.

   (c) Compute the error $\|\mathbf{y} - \mathbf{w}\|_\infty$ for each h. The error increases for not very small h because of boundary layers. After what h, the error starts decreasing?

3) [p. 672, Prob. 15(a)] A wooden beam of square cross section is supported at both ends and is carrying a distributed lateral load of intensity $w = 20\,\mathrm{lb/ft}$ and an axial tension load $T = 100\,\mathrm{lb}$. The deflection $u(x)$ of the beam's centerline satisfies

$$u'' - \frac{T}{EI}u = -\frac{w}{2EI}x(L-x), \qquad u(0) = u(L) = 0,$$

where $L = 6\,\mathrm{ft}$ is the length, $E = 1.3 \times 10^6\,\mathrm{lb/in^2}$ is the modulus of elasticity and $I = (4\,\mathrm{in})^4$ is the moment of inertia of the beam. Determine the deflection of the beam at 1-inch intervals along its length.