**Problem Set 8** (11/20, 22, 25, 27, 12/2)
**Due on Fri, Dec 6**

1) Consider the integral $\int_0^1 \mathrm{e}^{-x^2}\mathrm{d}x = 0.7468\ldots$. To numerically evaluate this integral we take $x_i = ih$, $h = 1/n$, $i = 0, 1, \ldots, n$.
   (a) Find the (right-hand) Riemann sum $R(h)$ for $h = 1, 0.5, 0.25, 0.125$.
   (b) Find the trapezoid rule $T(h)$ for $h = 1, 0.5, 0.25, 0.125$.
   (c) Find Simpson's rule $S(h)$ for $h = 1, 0.5, 0.25, 0.125$.
   (d) Find Richardson extrapolation (Romberg's method) $R_1(h)$ for $h = 0.5, 0.25, 0.125$.
   (e) Find Richardson extrapolation (Romberg's method) $R_2(h)$ for $h = 0.25, 0.125$.
   (f) Submit your codes.
2) We can construct Legendre polynomials by the Gram-Schmidt process starting from $\{1, x, x^2, \ldots\}$. Show all intermediate steps.
   (a) Find $P_2(x)$.
   (b) Find $P_3(x)$.
   (c) Find $P_4(x)$.
   (d) Find $P_5(x)$.