Let us consider $f(x) = \left(1+9x^2\right)^{-1}$.

$$\int_{-1}^{1} f(x)\mathrm{d}x = \frac{2}{3}\tan^{-1}(3) = 0.832697,$$
$$\int_{-1}^{1} p_2(x)\mathrm{d}x = \int_{-1}^{1}\left(-\frac{9}{10}x^2+1\right)\mathrm{d}x = \frac{7}{5} = 1.4.$$

Thus $p_2(x)$ is a poor approximation to $f(x)$. We try increasing n. We obtain

$$\int_{-1}^{1} p_4(x)\mathrm{d}x = 0.735385, \quad \int_{-1}^{1} p_8(x)\mathrm{d}x = 0.738204, \quad \int_{-1}^{1} p_{16}(x)\mathrm{d}x = 0.667583.$$

The approximations are still not good. For these Chebyshev points, we obtain $\int_{-1}^{1} p_2(x)\mathrm{d}x = 1.4$ and

$$\int_{-1}^{1} p_4(x)\mathrm{d}x = 1.00727, \quad \int_{-1}^{1} p_8(x)\mathrm{d}x = 0.844188, \quad \int_{-1}^{1} p_{16}(x)\mathrm{d}x = 0.832759.$$

Let us look at numerical results.