```
% -y''=r,  y(0)=alpha,  y(1)=beta
clear; clf;
alpha=0;
beta=1;
n=3;
h=1/(n+1);
x_exact=0:0.0025:1;
y_exact=25/pi^2*sin(pi*x_exact)+x_exact;
for i=1:n
   x(i)=i*h;
   y(i)=25/pi^2*sin(pi*x(i))+x(i);
   a(i)=-1/h^2;
   b(i)=2/h^2;
   c(i)=-1/h^2;
   r(i)=25*sin(pi*x(i));
end
r(1)=r(1)+alpha/h^2;
r(n)=r(n)+beta/h^2;
w=LU_factorization(a,b,c,r);
% output
table(1)=h;
table(2)=norm(y-w,inf);
table(3)=norm(y-w,inf)/h;
table(4)=norm(y-w,inf)/h^2;
table(5)=norm(y-w,inf)/h^3;
table
xplot=[0 x 1];
wplot=[alpha w beta];
plot(x_exact,y_exact,xplot,wplot,'g-',xplot,wplot,'ro')
axis([0 1 0 4])
title(sprintf('n=%d, h=1/%d',n,n+1),'FontSize',24)
set(gca,'FontSize',24)
```

```
function w = LU_factorization(a,b,c,r)
n=length(r);
u(1)=b(1);
for k=2:n
   l(k)=a(k)/u(k-1);
   u(k)=b(k)-l(k)*c(k-1);
end
z(1)=r(1);
```

```
for  k=2:n
   z(k)=r(k)-l(k)*z(k-1);
end
w(n)=z(n)/u(n);
for  k=n-1:-1:1
   w(k)=(z(k)-c(k)*w(k+1))/u(k);
end
```

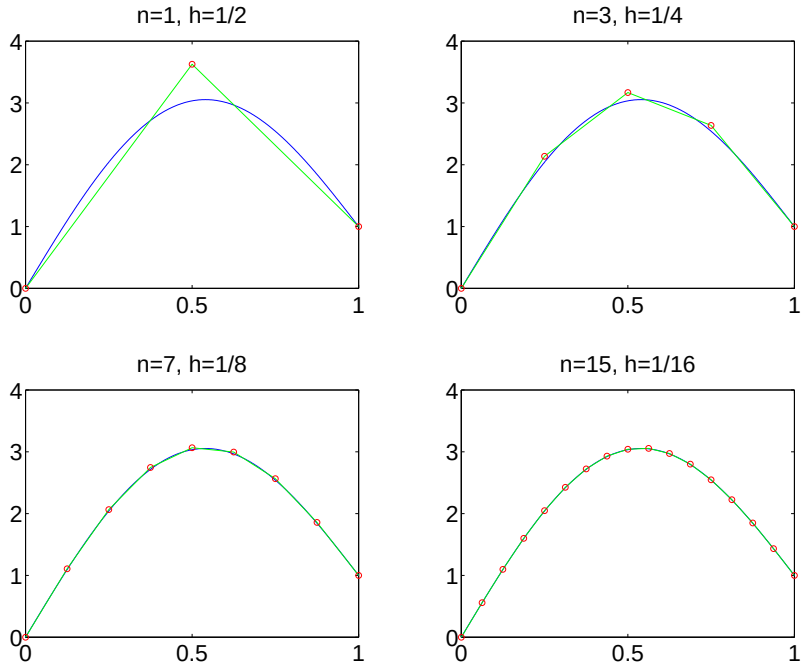

Fig. 2.1 Numerical solutions to $-y'' = 25\sin(\pi x), 0 \le x \le 1, y(0) = 0, y(1) = 1$. The exact solution is plotted as a solid curve.

h	$\|\mathbf{y}-\mathbf{w}\|_\infty$	$\|\mathbf{y}-\mathbf{w}\|_\infty/h$	$\|\mathbf{y}-\mathbf{w}\|_\infty/h^2$	$\|\mathbf{y}-\mathbf{w}\|_\infty/h^3$
0.5000	0.5920	1.1839	2.3679	4.7358
0.2500	0.1343	0.5373	2.1492	8.5968
0.1250	0.0328	0.2624	2.0995	16.7960
0.0625	0.0082	0.1305	2.0874	33.3977