Formulae

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}, \quad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$\cosh^2 x - \sinh^2 x = 1, \quad \cosh(-x) = \cosh x, \quad \sinh(-x) = -\sinh x$$
$$\cosh(2x) = \cosh^2 x + \sinh^2 x, \quad \sinh(2x) = 2\sinh x \cosh x, \quad \tanh(2x) = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\cosh^2 x = \frac{\cosh 2x + 1}{2}, \quad \sinh^2 x = \frac{\cosh 2x - 1}{2}, \quad 1 - \tanh^2 x = \operatorname{sech}^2 x = \frac{1}{\cosh^2 x}$$
$$\frac{d\cosh x}{dx} = \sinh x, \quad \frac{d\sinh x}{dx} = \cosh x, \quad \frac{d\tanh x}{dx} = \operatorname{sech}^2 x = \frac{1}{\cosh^2 x}$$

$$\begin{aligned} \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \end{aligned}$$

$$\begin{aligned} \cos A \cos B &= \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right] \\ \sin A \sin B &= \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right] \\ \sin A \cos B &= \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right] \\ \cos A \sin B &= \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right] \end{aligned}$$

$$\begin{aligned} \cosh(A \pm B) &= \cosh A \cosh B \pm \sinh A \sinh B \\ \sinh(A \pm B) &= \sinh A \cosh B \pm \cosh A \sinh B \\ \tanh(A \pm B) &= \frac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B} \end{aligned}$$

$$\begin{aligned} \cosh A \cosh B &= \frac{1}{2}\left[\cosh(A + B) + \cosh(A - B)\right] \\ \sinh A \sinh B &= \frac{1}{2}\left[\cosh(A + B) - \cosh(A - B)\right] \\ \sinh A \cosh B &= \frac{1}{2}\left[\sinh(A + B) + \sinh(A - B)\right] \\ \cosh A \sinh B &= \frac{1}{2}\left[\sinh(A + B) - \sinh(A - B)\right] \end{aligned}$$

Theorems

Theorem 1. For $m, n = 1, 2, \cdots$, we have

$$\int_{-L}^{L} \cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx = L\delta_{nm},$$
$$\int_{-L}^{L} \sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L}dx = L\delta_{nm},$$
$$\int_{-L}^{L} \sin\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx = 0.$$

Theorem 2. For $m, n = 1, 2, \cdots$, we have

$$\int_{0}^{L} \cos\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx = \frac{L}{2}\delta_{nm},$$
$$\int_{0}^{L} \sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L}dx = \frac{L}{2}\delta_{nm},$$
$$\int_{0}^{L} \sin\frac{n\pi x}{L}\cos\frac{m\pi x}{L}dx = \begin{cases} \dfrac{2Ln}{\pi\left(n^2 - m^2\right)} & \text{for odd } n+m, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 3. Consider the Sturm-Liouville problem

$$[s(x)\phi'(x)]' + [\lambda\rho(x) - q(x)]\phi(x) = 0, \quad a < x < b,$$

where $\rho(x) > 0$, with the boundary conditions

$$\phi(a)\cos\alpha - L\phi'(a)\sin\alpha = 0, \quad \phi(b)\cos\beta + L\phi'(b)\sin\beta = 0,$$

where $L = b - a$, and $\alpha, \beta \in [0, \pi)$ are some parameters. Suppose that $\phi_1(x), \phi_2(x)$ are nontrivial solutions with different eigenvalues $\lambda_1 \neq \lambda_2$. Then the eigenfunctions are orthogonal with respect to the weight function $\rho(x)$, $a < x < b$:

$$\int_a^b \phi_1(x)\phi_2(x)\rho(x)dx = 0.$$

Theorem 4. For $m, n = 1, 2, \cdots$, we have

$$\int_0^{L_2}\int_0^{L_1} \sin\frac{m\pi x}{L_1}\sin\frac{n\pi y}{L_2}\sin\frac{m'\pi x}{L_1}\sin\frac{n'\pi y}{L_2}dxdy = \frac{L_1 L_2}{4}\delta_{mm'}\delta_{nn'}.$$

Cylindrical and spherical coordinates

$$x = \rho\cos\varphi, \qquad y = \rho\sin\varphi, \qquad z = z,$$

$$x = r\sin\theta\cos\varphi, \qquad y = r\sin\theta\sin\varphi, \qquad z = r\cos\theta.$$

Bessel's equation

$$J_m''(x) + \frac{1}{x}J_m'(x) + \left(1 - \frac{m^2}{x^2}\right)J_m(x) = 0.$$

The Legendre equation

$$\left[(1-s^2)P_k'(s)\right]' + k(k+1)P_k(s) = 0.$$

Fourier transform

$$f(x) = \int_{-\infty}^{\infty} \tilde{f}(\mu)e^{i\mu x}d\mu, \qquad \tilde{f}(\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\mu x}dx.$$

Gaussian integral

$$\int_{-\infty}^{\infty} e^{-a(x-b)^2}dx = \sqrt{\frac{\pi}{a}}, \qquad a > 0.$$

Green's functions

The solution to

$$\begin{cases} u_t - Ku_{xx} = h(x,t), & t > 0, \quad -\infty < x < \infty, \\ u = f(x), & t = 0, \quad -\infty < x < \infty, \end{cases}$$

is given by

$$u(x,t) = \int_{-\infty}^{\infty} G(x,x';t)f(x')dx' + \int_0^t \int_{-\infty}^{\infty} G(x,x';t-s)h(x',s)dx'ds,$$

where

$$G(x,x';t) = \frac{1}{\sqrt{4\pi Kt}}e^{-(x-x')^2/(4Kt)}.$$