Formulae

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}, \quad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1, \quad \cosh(-x) = \cosh x, \quad \sinh(-x) = -\sinh x$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x, \quad \sinh(2x) = 2\sinh x \cosh x, \quad \tanh(2x) = \frac{2\tanh x}{1 + \tanh^2 x}$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}, \quad \sinh^2 x = \frac{\cosh 2x - 1}{2}, \quad 1 - \tanh^2 x = \operatorname{sech}^2 x = \frac{1}{\cosh^2 x}$$

$$\frac{d\cosh x}{dx} = \sinh x, \quad \frac{d\sinh x}{dx} = \cosh x, \quad \frac{d\tanh x}{dx} = \operatorname{sech}^2 x = \frac{1}{\cosh^2 x}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$$

$$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$$

$$\tanh(A \pm B) = \frac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$$

$$\cosh A \cosh B = \frac{1}{2}\left[\cosh(A + B) + \cosh(A - B)\right]$$

$$\sinh A \sinh B = \frac{1}{2}\left[\cosh(A + B) - \cosh(A - B)\right]$$

$$\sinh A \cosh B = \frac{1}{2}\left[\sinh(A + B) + \sinh(A - B)\right]$$

$$\cosh A \sinh B = \frac{1}{2}\left[\sinh(A + B) - \sinh(A - B)\right]$$

Theorems

Theorem 1. For $m, n = 1, 2, \cdots$, we have

$$\int_{-L}^{L} \cos \frac{n\pi x}{L} \cos \frac{m\pi x}{L} dx = L\delta_{nm},$$

$$\int_{-L}^{L} \sin \frac{n\pi x}{L} \sin \frac{m\pi x}{L} dx = L\delta_{nm},$$

$$\int_{-L}^{L} \sin \frac{n\pi x}{L} \cos \frac{m\pi x}{L} dx = 0.$$

Theorem 2. For $m, n = 1, 2, \cdots$, we have

$$\int_{0}^{L} \cos \frac{n\pi x}{L} \cos \frac{m\pi x}{L} dx = \frac{L}{2}\delta_{nm},$$

$$\int_{0}^{L} \sin \frac{n\pi x}{L} \sin \frac{m\pi x}{L} dx = \frac{L}{2}\delta_{nm},$$

$$\int_{0}^{L} \sin \frac{n\pi x}{L} \cos \frac{m\pi x}{L} dx = \begin{cases} \dfrac{2Ln}{\pi(n^2 - m^2)} & \text{for odd } n + m, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 3. Consider the Sturm-Liouville problem

$$[s(x)\phi'(x)]' + [\lambda\rho(x) - q(x)]\phi(x) = 0, \quad a < x < b,$$

where $\rho(x) > 0$, with the boundary conditions

$$\phi(a)\cos\alpha - L\phi'(a)\sin\alpha = 0, \quad \phi(b)\cos\beta + L\phi'(b)\sin\beta = 0,$$

where $L = b - a$, and $\alpha, \beta \in [0, \pi)$ are some parameters. Suppose that $\phi_1(x), \phi_2(x)$ are nontrivial solutions with different eigenvalues $\lambda_1 \neq \lambda_2$. Then the eigenfunctions are orthogonal with respect to the weight function $\rho(x)$, $a < x < b$:

$$\int_{a}^{b} \phi_1(x)\phi_2(x)\rho(x)dx = 0.$$

Theorem 4. For $m, n = 1, 2, \cdots$, we have

$$\int_{0}^{L_2} \int_{0}^{L_1} \sin \frac{m\pi x}{L_1} \sin \frac{n\pi y}{L_2} \sin \frac{m'\pi x}{L_1} \sin \frac{n'\pi y}{L_2} dx dy = \frac{L_1 L_2}{4}\delta_{mm'}\delta_{nn'}.$$