Inverse problems for graphs and discrete spaces

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We study the inverse problem of determining a finite weighted graph (X, E) from the source-to-solution map on a vertex subset $B \subset X$ for heat equations on graphs, where the time variable can be either discrete or continuous. We prove that this problem is equivalent to the discrete version of the inverse interior spectral problem, provided that there does not exist a nonzero eigenfunction of the weighted graph Laplacian vanishing identically on B. In particular, we consider inverse problems for discrete-time random walks on finite graphs. We show that under the Two-Points Condition, the graph structure and the transition matrix of the random walk can be uniquely recovered from the distributions of the first passing times on B, or from the observation on B of one realization of the random walk. In addition, we will consider the problem of approximating discrete metric spaces or graphs by smooth manifolds.